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Analysis of DOA Estimation for Isotropic Antenna using Music and ESPIRIT Algorithm

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Abstract: The deployment of smart antennas provides greater capacity and performance benefits over standard antennas because they are used to customize antenna coverage patterns to match the traffic conditions in a wireless network. The two basic functions of any smart antenna are Direction of Arrival (DOA) Estimation and Adaptive Beam forming (ABF). Direction of Arrival algorithms work on the signals received at the antenna array elements and computes the angle of arrival of all incoming signals that include the desired user signal, interfering signals and multipath signals. Ultimately a beam is steered along the direction of the desired signal, while a null is generated along the direction of the interfering signal. The smart antenna sub system estimate the direction of arriving signals by employing DOA algorithms. The performance of smart antenna greatly depends on the effectiveness of DOA estimation algorithm. This paper analyzed the performance of MUSIC (Multiple Signal Classification) and ESPIRIT (Estimation of Signal Parameters via Rotational Invariance Technique) algorithms for DOA estimation. All the simulations are carried out using MATLAB.

Keywords: DOA, MATLAB, ESPRIT, ABF

I. INTRODUCTION

Smart antennas systems are being the key solution to increase the spectral efficiency and improving the system performance in mobile communication. Smart antennas usually consist of a number of radiating elements known as array antennas whose individual excitation can be controlled in order to achieve the desired radiation pattern. The smart antennas systems estimate the direction of arrival of the signal using different techniques. They involve finding a spatial spectrum of the antenna array, and calculating the DOA from the peaks of this spectrum. These calculations are computationally intensive.

In case of linear array antenna, the DOA is specified by one angle. To find the corresponding DOA, subspace method algorithms like Multiple Signal Classification (MUSIC) and ESPIRIT (Estimation of Signal Parameters via Rotational Invariance Technique) is considered these algorithms are simulated for 4-element linear array and 8-element linear array. In case of planar array antenna, the DOA is specified by two separate angles that are azimuth angle and elevation angle. MUSIC algorithm is dealt with for a 4x4 and 6x6 planar array. The MUSIC algorithm is tested based on certain parameters like its performance for different Signal to Noise Ratio (SNR), the resolving capability and its performance for different snapshots. The organization of paper is as follows. Section 2 gives introduction. Section 3 gives brief overview of MUSIC algorithm.

Section 4 gives brief overview of ESPIRIT algorithm Section 5 gives simulated results for MUSIC. Section 5 gives simulated results for ESPIRIT. Section 6 gives the conclusion.

II. INPUT DATA MODEL

In more general case as shown in Fig.1 the input data model of multiple signals and interference impinging on the array by decomposing the input signal in frequency domain using linear superposition.

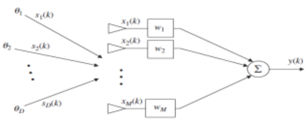

Fig. 1: Input data model

$$x(\omega) = \sum_{i=1}^{D} a(\omega, \theta_i, \phi_i)\, s_i(\omega) + n(\omega) \quad (1)$$

Using matrix notation, this can be represented as

$$x(\omega) = \begin{bmatrix} a(\omega, \theta_i, \phi_i) & \ldots & a(\omega, \theta_D, \phi_D) \end{bmatrix} \begin{bmatrix} s_i(\omega) \\ \ldots \\ s_D(\omega) \end{bmatrix} + n(\omega) \quad (2)$$

or

$$x(\omega) = A(\omega, \Theta, \Phi)\, s(\omega) + n(\omega) \quad (3)$$

where D signals $S_j(w)$,……$S_D(w)$ arrive from angles (θ_i,ϕ_i),…(θ_D,ϕ_D) respectively and n(w) represents the interference and noise components.

If the frequency band of interest is sufficiently narrow to categorize the array as a narrowband array, the dependence on w can be dropped and the array data can be modeled in time domain as the analytic signal.

$$x(t) = \sum_{i=1}^{D} a(\theta_i, \phi_i)\, s_i(t) + n(t) \quad (4)$$

III. MUSIC ALGORITHM

Within the class of the so-called signal subspace algorithms, MUSIC has been the most widely examined. MUSIC stands for Multiple Signal Classification. Schmidt developed the MUSIC algorithm by noting that the desired signal array response is orthogonal to the noise subspace. MUSIC algorithm involves the decomposition of the covariance matrix into two orthogonal matrices i.e., signal subspace and noise subspace assuming that noise is highly uncorrelated. The covariance matrix in terms of eigenvalues and eigenvectors is

$$\mathbf{R}_{xx} = \sum_{n=1}^{M} \lambda_n \mathbf{e}_n \mathbf{e}_n^H = \mathbf{E}_s \Lambda_s \mathbf{E}_s^H + \mathbf{E}_n \Lambda_n \mathbf{E}_n^H$$

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Where
$E_s = [e_1, e_2, \ldots, e_K]$ and $E_n = [e_{K+1}, e_{K+2}, \ldots, e_{M*N}]$ are the signal Eigen vector matrix and noise Eigen vector matrix respectively. $\Lambda_s = \text{diag}\ \{\lambda_1, \lambda_2, \ldots, \lambda_K\}$ and $\Lambda_n = \text{diag}\ \{\lambda_{K+1}, \lambda_{K+2}, \ldots, \lambda_{M*N}\}$ are the signal Eigen value matrix and noise Eigen value matrix respectively. The subspaces are determined, the DOAs of the desired signals can be estimated by calculating the MUSIC spatial spectrum over the region of interest by

$$P_M(\theta) = \frac{1}{\bar{a}^H(\theta).\bar{E}_N.\bar{E}_N^{\ H}.\bar{a}(\theta)}$$

IV. ESPIRIT

ESPRIT stands for Estimation of Signal Parameters via Rotational Invariance Techniques and was first proposed by Roy and Kailath in 1989. The goal of the ESPRIT technique is to exploit the rotational invariance in the signal subspace which is created by two arrays with a translational invariance structure.

ESPRIT inherently assumes narrowband signals so that one knows the translational phase relationship between the multiple arrays to be used. As with MUSIC, ESPRIT assumes that there are D < M narrow-band sources centered at the center frequency f_0 .

These signal sources are assumed to be of a sufficient range so that the incident propagating field is approximately planar. The sources can be either random or deterministic and the noise is assumed to be random with zero-mean. ESPRIT assumes multiple identical arrays called doublets.

These can be separate arrays or can be composed of subarrays of one larger array. It is important that these arrays are displaced transnationally but not rotationally. An example is shown in Fig 2. Where a four element linear array is composed of two identical three-element sub arrays or two doublets. These two sub arrays are transnationally

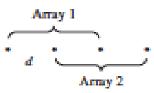

Fig 2:Doublet composed of two identical displaced arrays.

IV. SIMULATED RESULTS FOR MUSIC

1. Analysis of MUSIC for 4-element and 8-element linear Array antenna

1.1 Varying SNR values:

MUSIC algorithm is simulated for a linear array antenna of 4-elements with angles 30^o and 35^o for SNR values 5dB, 10dB and 15dB respectively as shown in Fig. 3. The same thing is been done for 8-element linear array in Fig. 4. All the three SNR values considered are giving better peaks, this is where Sub-space methods are having far better performance than Traditional methods of DOA

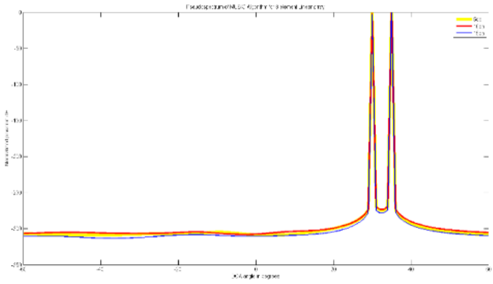

Fig. 3: Varying SNR plot for 4-element linear array

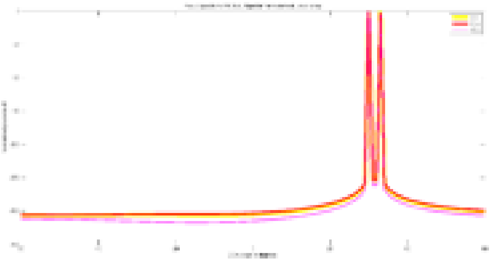

Fig. 4: Varying SNR plot for 8-element linear array

1.2 Resolution of algorithm

The simulation of MUSIC algorithm for a linear array of 4-elements and 8-elements with a constant SNR of 5dB is shown in Fig. 5 and Fig. 6. In both cases, it is found that angles 0^o and 1^o (1^o resolution) are not resolved, angles -35^o and -33^o (2^o resolution) are just resolved and angles 30^o and 33^o (3^o resolution) are well resolved. Thus at a given SNR of 5dB, there is not much difference between the performance of 8-element linear array and 4-element linear array.

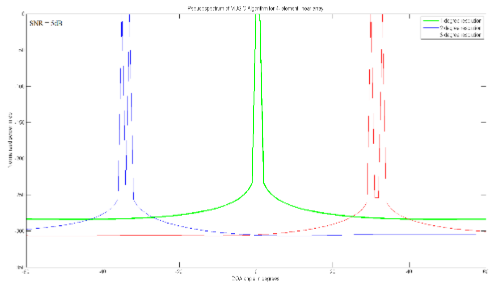

Fig. 5: Resolution study for 4-element linear array

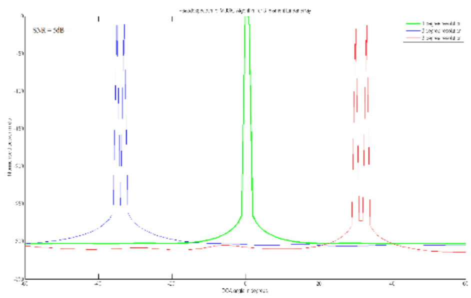

Fig. 6: Resolution study for 8-element linear array

1.3 Varying number of Snap shots

MUSIC algorithm is simulated for a linear array antenna of 4-elements with angles 30^o, 50^o for SNR value of 5dB

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for k=3,5,7,9 as shown in Fig. 7. The same is also done for linear array of 8-elements in Fig. 8. It is observed that in both the plots better peaks are obtained for high snapshots. It is also observed that better peaks are obtained for 8-element array than 4-element array. Hence the number of elements in an array is a measure of its performance.

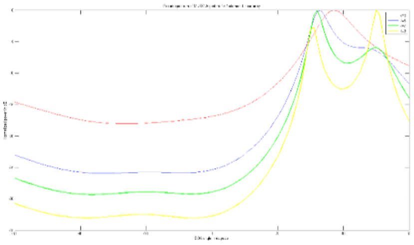

Fig. 7: Varying snap shot study for 4-element linear array

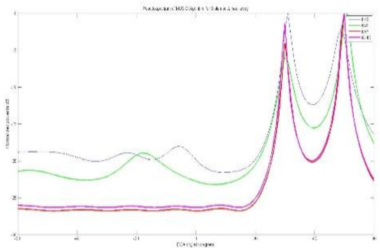

Fig. 8: Varying snap shot study for 8-element linear array

2. Analysis of MUSIC for Planar Array Antennas

The performance of the algorithms are analyzed for the cases:

1. The performance of algorithm is analyzed for different SNR values
2. The minimum angular separation that can be allowed between two sources so that the algorithm can resolve them i.e., Resolution of the algorithm is determined.

In Planar Array the Direction of arrival of a signal is specified by Elevation angle and Azimuthal angle. Hence planar array antenna should be analyzed in two different planes i.e., elevation plane and azimuthal plane.

2.1 Analysis for 4x4 Planar Array antenna

2.1.1 Varying SNR

MUSIC algorithm is applied for 4X4 planar array for azimuthal angles 15^o and 40^o with constant elevation angle of 20^o for 5dB, 10dB and 15dB SNR values. As SNR value increases, it is observed that better peaks are obtained. At 5dB SNR, better peak is obtained compared to that of at 10dB and 15dB respectively as shown in Fig. 9. So the SNR value of incoming signal is a measure of performance of algorithm.

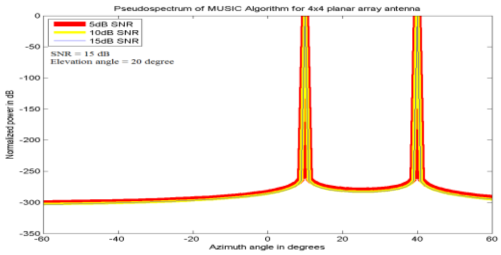

Fig. 9: Varying SNR plot of azimuthal variation (15^o and 40^o) with constant elevation angle of 20^o for 4X4 planar array

MUSIC algorithm is applied for 4X4 planar array for elevation angles 5^o and 50^o with constant azimuthal angle of 10^o for 5dB, 10dB and 15dB SNR values. As SNR value increases it is observed that better peaks are obtained. At 15dB SNR better peak is obtained compared to that of at 5dB and 10dB respectively as shown in Fig. 10 .

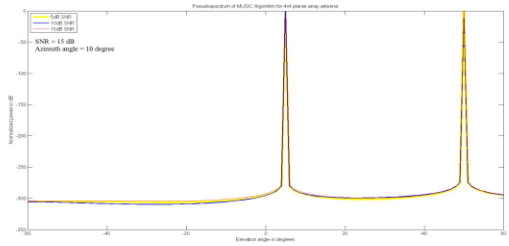

Fig. 10: Varying SNR plot of elevation variation (5^o and 50^o) with constant azimuthal angle of 10^o for 4X4 planar array

2.1.2 Resolution of algorithm:

The resolution of MUSIC algorithm in azimuthal plane with a constant elevation angle of 20^o at a given SNR of 15dB is analyzed as shown in Fig. 11. It is found that azimuthal angles -34^o & -35^o (1^o separation) are not resolved, 0^o& 2^o (2^o separation) are just resolved and 30^o & 33^o (4^o separation) are well resolved. Therefore it can be said that at a SNR of 15dB MUSIC algorithm can resolve 3^o angular separation in azimuthal plane.

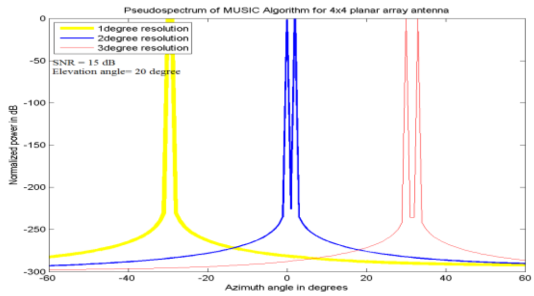

Fig. 11: Resolution study of MUSIC algorithm for 4X4 planar array in Azimuthal plane

The resolution of MUSIC algorithm in elevation plane with a constant azimuthal angle of 10^o at a SNR of 15dB is analyzed as shown in Fig. 12. It is found elevation angles -34^o & -35^o (1^o separation) is just resolved, whereas 0^o & 2^o (2^o separation) and 30^o & 35^o (3^o separation) are well resolved. Therefore it can be said that at a SNR of 15dB MUSIC algorithm can resolve 3^o angular separation in azimuthal plane.

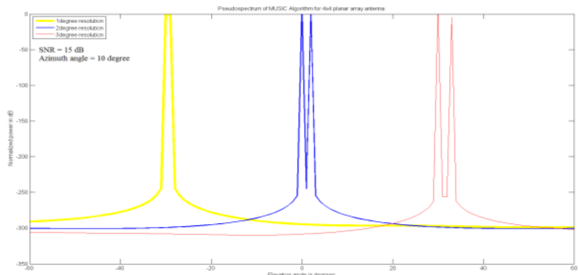

Fig. 12: Resolution study of MUSIC algorithm for 4X4 planar array in elevation plane

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2.2 Analysis of MUSIC for 6x6 Planar Array antenna

2.2.1 Varying SNR values

MUSIC algorithm is applied for 6x6 planar array for azimuthal angles 10^o and 40^o with constant elevation angle of 20^o for 5dB, 10dB and 15dB SNR values. As SNR value increases, it is observed that better peaks are obtained. At 5dB SNR, better peak is obtained compared to that of at 10dB and 15dB respectively as shown in Fig. 13. So the SNR value of incoming signal is a measure of performance of algorithm.

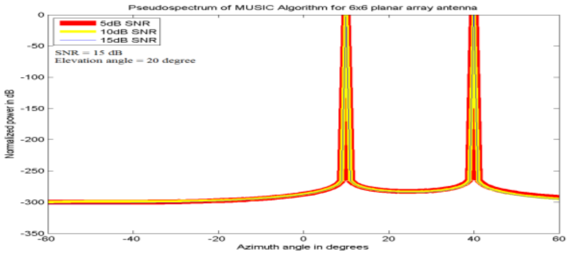

Fig. 13: Varying SNR plot of azimuthal variation (10^o and 40^o) with constant elevation angle of 20^o for 6x6 planar array

MUSIC algorithm is applied for 6x6 planar array for elevation angles 5^o and 50^o with constant azimuthal angle of 10^o for 5dB, 10dB and 15dB SNR values. As SNR value increases it is observed that better peaks are obtained. At 15dB SNR better peak is obtained compared to that of at 5dB and 10dB respectively as shown in Fig. 14.

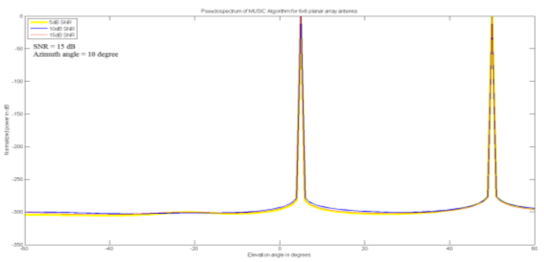

Fig. 14: Varying SNR plot of elevation variation (5^o and 50^o) with constant azimuthal angle of 10^o for 6x6 planar array

2.2.2 Resolution of algorithms

The resolution of MUSIC algorithm in azimuthal plane with a constant elevation angle of 20^o at a given SNR of 15dB is analyzed as shown in Fig. 15. It is found that azimuthal angles -34^o & -35^o (1^o separation) are not resolved, 0^o& 2^o (2^o separation) are just resolved and 30^o & 33^o (4^o separation) are well resolved. Therefore it can be said that at a SNR of 15dB MUSIC algorithm can resolve 3^o angular separation in azimuthal plane.

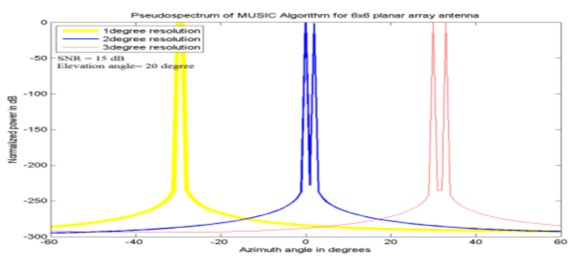

Fig. 15: Resolution study of MUSIC algorithm for 6x6 planar array in Azimuthal plane

The resolution of MUSIC algorithm in elevation plane with a constant azimuthal angle of 10^o at a SNR of 15dB is analyzed as shown in Fig. 16. It is found elevation angles -34^o & -35^o (1^o separation) is just resolved, whereas 0^o & 2^o (2^o separation) and 30^o & 35^o (3^o separation) are well resolved. Therefore it can be said that at a SNR of 15dB MUSIC algorithm can resolve 3^o angular separation in azimuthal plane.

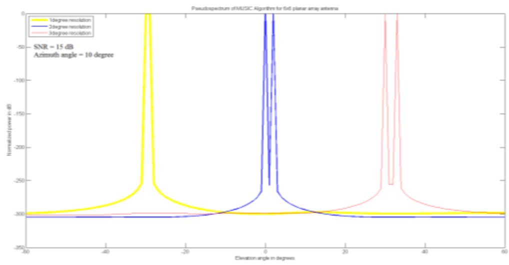

Fig. 16: Resolution study of MUSIC algorithm for 6x6 planar array in elevation plane

2.2.3 Varying number of Snap shots

MUSIC algorithm is simulated for a linear array antenna of 4X4 planar array-elements with angles 5^o, 50^o for SNR value of 5dB for k=3,5,7,9 as shown in

Fig. 17. The same is also done for linear array of 6X6 planar array-elements in Fig. 18. It is observed that in both the plots better peaks are obtained for high snapshots. It is also observed that better peaks are obtained for 6X6 planar array-elements than 4X4 planar array-elements. Hence the number of elements in an array is a measure of its performance

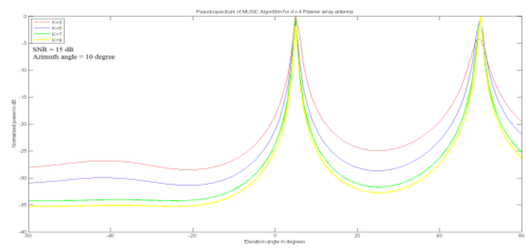

Fig. 17: Varying snapshot study of MUSIC algorithm for 6x6 planar array in elevation plane

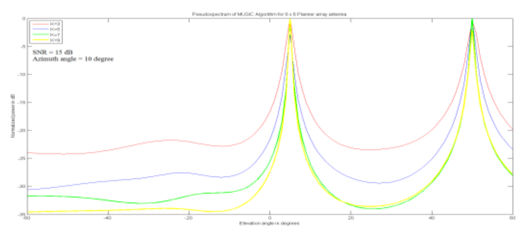

Fig 18: Varying snap shot study of MUSIC algorithm for 6x6 planar array in elevation plane

3. Probability of resolution and RMSE

Probability of resolution $P_{res}=P_r\{\Upsilon>0\}$,where

$$g(q_1,q_2)=P(q_m)-\frac{1}{2}\{P(q_1)+P(q_2)\}$$

Where θ_1,θ_2 are the two angle of arrival and θ_m is the average of both angles. Fig 19 shows the plot of

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probability of resolution versus ASNR. Fig 20 shows the plot of RMSE versus SNR plot.

RMSE is the square root of the mean/average of the square of all of the error. The use of RMSE is very common and it makes an excellent general purpose error metric for numerical predictions. Compared to the similar Mean Absolute Error, RMSE amplifies and severely punishes large errors.

$$RMSE=\sqrt{\frac{1}{T}\sum_{i=0}^{T}\left(\theta-\hat{\theta}\right)^{2}}$$

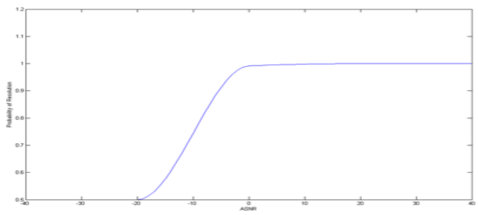

Fig 19: Probability of resolution vs ASNR.

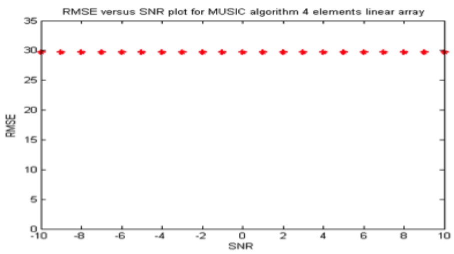

Fig 20: RMSE versus SNR plot.

V. SIMULATED RESULTS OF ESPIRIT

1. Linear array

The simulation of ESPRIT algorithm for a linear array of 8-elements with each sub array having 4 elements is done for two methods. Table 1 shows results for Total Least Square ESPRIT, and Table 2 shows the results for Least Square ESPRIT with interspacing distance between sub arrays being 1 element a constant SNR of 15.22 dB .

Source 1 DOA (degrees)	Source 2 DOA (degrees)	Source 1 Estimated DOA(degrees)	Source 2 Estimated DOA(degrees)
10	15	8.07	13.94
20	26	22.12	29.3
15	30	14.78	30.65
40	52	40.9	71.2
40	60	40.71	59.26

Table 1: Total Least Square ESPRIT

Source 1 DOA (degrees)	Source 2 DOA (degrees)	Source 1 Estimated DOA(degrees)	Source 2 Estimated DOA(degrees)
10	15	10.55	13.22
20	26	17.41	24.89
15	30	14.27	28.51
40	52	41.76	60.53
40	60	41.19	65.26

Table 2: Least Square ESPRIT

2. Planar array

The simulation of ESPRIT algorithm for a 4x4 planar array of with each sub array having 12 elements (3x4). Table 3 shows the results for Least Square ESPRIT with interspacing distance between sub arrays being 1 element a constant SNR of 15dB.

S1 azimuth angle	S1 elevation angle	S2 azimuth angle	S2 elevation angle
10	15	30	40
20	26	50	56
15	30	30	50
40	50	60	70

Table 3: Least Square ESPRIT

Table 4 shows the estimated values for both azimuthal and elevation plane.

S1 Estimated azimuth angle	S2 Estimated Elevation angle	S2 Estimated Azimuth Angle	S2 Estimated Elevation Angle
9.73	15.06	29.32	40.44
19.81	25.42	49.08	54.19
13.79	28.52	28.74	50.82
39.55	48.1	58.37	68.44

Table 4 : Estimated values for Least square ESPIRIT

VI. CONCLUSIONS

The simulation result of Music algorithm shows that angular resolution of Music algorithm improves with more no. of elements in the array, with large snapshot of signals and greater angular separation between the signals. MUSIC can estimate uncorrelated signal very well but it fails to detect correlated signals. ESPIRIT algorithm also shows the better results.

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